Mathematics/Logical Thinking Test Specification

May 2024



Contents

1	Introduction	. 3
2	Structure of the mathematics test	. 3
3	Rules for selecting candidates for admission	. 3
4	Requirements	. 4

1 Introduction

This document contains important information and guidance for candidates preparing for the July 2024 New Uzbekistan University (NewUU) admissions test in Mathematics and Logical Thinking. Details of the test, subject content and sample items are all included.

Cambridge (Cambridge University Press and Assessment) have designed and produced this test in association with the New Uzbekistan University admissions unit. The test is designed to assess mathematical skills and logical thinking abilities. The results from the test will be used by NewUU to select candidates for entry on their courses and will be published on Cambridge's website on July/August 2024.

2 Structure of the mathematics and logical thinking test

The mathematics test will consist of a single 120-minute question paper.

The paper will consist of a total of 40 multiple choice questions. Questions will be divided into two sections. Section A will consist of 34 Maths questions and Section B will consist of 6 questions assessing logical thinking.

All questions will be worth 1 mark and there will be no penalty for incorrect answers, so candidates are advised to answer **all** questions in the paper.

Each question may assess more than one topic. For example, a question about geometry may involve surds.

There is no formulae booklet for this test; candidates are expected to understand and recall all relevant formulae.

Calculators, mobile phones and any other electronic devices and unauthorised materials are **not** allowed.

3 Rules for selecting candidates for admission

If there are more applicants to the New Uzbekistan University than there are places available, the admissions unit of the university will carry out a selection process. The results from the admissions test will be the primary method of selecting applicants for entry to their chosen course, however NewUU may decide to use other evidence as well.

4 Requirements

Candidates are advised to read through these specifications carefully to ensure they are aware of all topics and areas that could be covered in the test.

PROBABILITY AND STATISTICS

Content area		Candidates may be tested on their ability to:
Sample space and events	1.1.	Evaluate probabilities in simple cases by means of enumeration of equally-likely elementary events.
Probabilities	1.2	Calculate the probability of simple combined events and conditional probability using possibility diagrams, tree diagrams and Venn diagrams.
Binomial probabilities	1.3	Calculate probabilities for the binomial distribution.
Conditional Probability and Independence	1.4	Calculate conditional probability. Determine whether or not events are independent.
Calculate measures of central tendency and dispersion	1.5	Calculate and use measures of tendency (mean, median, mode) and dispersion (range, interquartile range, standard deviation and variance) and distinguish between the purposes for which they are used.
Interpret data from tables	1.6	Interpret information given in line graph, bar chart, pie chart and other simple representations.
Permutations	1.7	Solve problems about arrangements of objects in a line, including those involving: • repetition (e.g. the number of ways of arranging the letters of the word APPLE) • restriction (e.g. the number of ways of arranging the letters of the word APPLE if all the vowels must be together).
Combinations	1.8	Solve problems involving selection.

These sample items are provided to give an indication of the types of items that will be used in the test. They are not intended to be a comprehensive description of all of the types of items that will be used.

- 1. How many ways are there to arrange the letters in the word APPLE?
 - A. 5
 - B. 10
 - C. 24
 - D. 60

Key: D

2. A boy flips a fair coin 6 times.

Find the probability that he gets exactly 3 heads.

- A. $\frac{5}{16}$
- B. $\frac{1}{64}$
- C. $\frac{1}{2}$
- D. $\frac{3}{4}$

Key: A

3. The median of these four numbers is equal to their mean.

χ,

10,

20,

12

Find the sum of all the possible values of x.

- A. 18
- B. 20
- C. 24
- D. 42

Key: D

POLYNOMIALS

Polynomials	2.1	Expand brackets involving polynomials. Factorise expressions. Algebraic simplification and manipulation.
Roots of polynomials	2.2	Solve linear, quadratic and cubic equations. Know and use quadratic formula.
Inequalities	2.3	Solve linear and quadratic inequalities. Solve simultaneous inequalities.
Simultaneous equations	2.4	Solve systems of equations (two linear or one linear, one quadratic).
Polynomial division	2.5	Divide a polynomial by a polynomial. Use the terms quotient and remainder.

- 1. Find the product of $(x^2 2)^2$ and $x^2 + 2$.
 - A. $x^4 4x^2 + 4$
 - B. $x^4 4$
 - C. $x^6 2x^4 8$
 - D. $x^6 2x^4 4x^2 + 8$

Key: D

- 2. Find the remainder when dividing $x^4 2x^3 2x + 1$ by $x^2 3x + 2$.
 - A. -1-x
 - B. 2-x
 - C. 1 3x
 - D. -x + 3

Key: A

3. The simultaneous equations

$$y = \frac{5}{2} - \frac{x}{2}$$

$$y = x^2 - \frac{k}{2}x + \frac{13}{2}$$

where k > 0 is a constant, have a unique solution x = m, y = n.

Find the value of m + n + k.

- A. 10.5
- B. 12.5
- C. 15
- D. 17

Key: B

GEOMETRY

Classification of triangles by sides	3.1	Know the different types of triangle (right-angled, equilateral, scalene, isosceles) and use their properties in calculations. Know and use Pythagoras' theorem.
Classification of triangles by angles	3.2	Use sine, cosine and tangent in a right-angled triangle. Know and use exact trigonometric ratios for common angles. Work with angles measured in degrees or radians. Use the formulae for the area of triangle as ½ base x height and ½ absinC.
Angle properties	3.3	Solve problems with angles involving lines, triangles, circles, polygons. Know and use symmetry of shapes.
Quadrilaterals	3.4	Know the properties of different types of quadrilateral. Know and use formulae for areas and perimeters.
Circles	3.5	Understand the terms radius, diameter, chord, arc, sector. Calculate the circumference and area of a circle. Know that the tangent is perpendicular to the radius in a circle. Know and use symmetry. Solve problems using length of arc and area of sector.
Composite shapes	3.6	Solve problems using perimeters and areas of composite shapes.

1. A circle has an area of 16π square units.

Find the length of the diameter.

- A. 8
- B. 16
- C. $4\sqrt{2}$
- D. $4\sqrt{\pi}$

Key: A

- 2. Find the maximal possible area of a triangle with two sides equal to 7 and 4.
 - A. $7\sqrt{2}$
 - B. 14
 - C. $14\sqrt{2}$
 - D. 28

Key: B

- **3.** A triangle with sides of length 6 cm, 8 cm and 10 cm has an altitude drawn to the longest side. Find the length of this altitude.
 - A. 4
 - B. 4.8
 - C. 5
 - D. 5.4

Key: B

FUNCTIONS

Types of function	4.1	Know what is meant by functions that are linear, quadratic, polynomial, rational, exponential (a^x) , logarithmic or trigonometric. Solve problems involving odd and even functions.
Domain and range	4.2	Understand the terms domain and range and know how to find them in problems involving functions.
Function Arithmetic	4.3	Use addition, subtraction, multiplication, division of functions.
Function Composition	4.4	Solve problems involving composition of two functions.
Inverse functions	4.5	Know when the inverse of a function exists. Find the inverse of a given function.

1. The function $f(x) = \frac{2x-1}{x+3}$

Find $f^{-1}(x)$.

- A. $f^{-1}(x) = \frac{x+3}{2x-1}$
- B. $f^{-1}(x) = \frac{x-3}{2x+1}$
- C. $f^{-1}(x) = \frac{x+1}{2x-3}$
- D. $f^{-1}(x) = \frac{3x+1}{2-x}$

Key: D

- 2. Find the domain of the function $f(x) = \frac{x+1}{\sqrt{x^2-9}}$
 - A. $x \in (-\infty, -3) \cup (3, \infty)$
 - B. $x \in (-\infty, 3) \cup (3, \infty)$
 - C. $x \in [-3, 3]$
 - D. $x \in (3, \infty)$

Key: A

- **3.** Which of the following functions is odd?
 - A. $x^2 + 2$
 - B. $\frac{1}{2x}$
 - C. $ee^x + ee^{-x}$
 - D. $\sqrt{x^2 + 1}$

Key: B

SERIES, POWERS, ROOTS

Arithmetic progression	5.1	Solve problems involving <i>n</i> th term and sum of <i>n</i> terms.	
Geometric progression	5.2	Solve problems involving <i>n</i> th term and sum of <i>n</i> terms (not the sum to infinity).	
Recursive sequences	5.3	Use formulae, for example, with a_{n+1} in terms of a_n	
Powers	5.4	Simplify algebraic expressions involving powers (integer, rational, negative).	
Roots	5.5	Simplify of algebraic expressions involving roots (integer, rational, negative).	
		Simplify expressions involving surds, including rationalising the denominator.	

Sample Questions

- 1. Find the sum of the first 20 terms of an arithmetic progression where the first term is 5 and the common difference is 3.
 - A. 590
 - B. 620
 - C. 670
 - D. 700

Key: C

- 2. Find the value of $a^4 + b^4$ if a and b are the roots of the quadratic equation $x^2 + 3x 5 = 0$
 - A. 212
 - B. 526
 - C. 311
 - D. 421

Key: C

LOGICAL THINKING

Logical thinking is the ability to analyse information and work out not just what it means, but what it entails. In other words, what would follow from it, supposing it were true.

The value of logical thinking is to allow systematic thinking and establish what can be deduced or inferred from given information. It is useful for solving problems and identifying where errors are made in thinking.

Within the broad category of logical thinking, there are a number of sub-skills:

- Understand logical connectives.
- Understand and successfully apply the rules of conditional statements.
- Make logical deductions.
- Identify what conclusions can reliably be drawn from given information, and with what degree of strength or confidence (inductive reasoning).
- Use notions of consistency and inconsistency to identify contradictions and implications.

Logical thinking questions often involve two or more of these skills. For example, any question that requires understanding of conditional statements, and the rules that govern these, will necessarily also test either deductive or inductive reasoning skills.

Logical connectives

An understanding of logical connectives is needed to understand what does or does not follow from information presented. The key logical connectives are 'and', 'or', 'not' and 'if...then'. They can appear alone or in combination.

Example 1: 'Billy does not have two children and a dog' means the statement 'Billy has two children and a dog' is *not* true. From this, the only inference that can be made is that Billy does not have two children *and* a dog. He might have two children but no dog. Or he might have a dog but no children. Or he might have neither two children nor a dog.

Example 2: 'Either Billy has two children and a dog or Samantha is from Switzerland'. In this case, if one of these claims is true, then the other one cannot be.

'If... then' connectives, otherwise known as conditional statements, are explained in more detail below.

Conditional statements

A statement in the form 'If x then y' is known as a conditional statement. It gives information about the relationship between x and y but, on its own, does not give information about whether x or y are actually true.

For example, the statement 'If Monika arrives before 3 pm then she will be in time to see the concert' provides information about the relationship between arriving before 3 pm and being in time to see the concert. It does not give information about when Monika actually arrives, nor whether she is actually in time to see the concert. It simply states that *if* it's true that Monika arrives before 3pm *then* it must also be true that she will be in time to see the concert.

However, if there is additional information given about either *x* or *y* being true or false independently, further deductions can be made.

1. From conditional statement 'If x then y' and information that x is true, it can be inferred that y must also be true.

The statement 'If x then y' implies that if x is true, y must also be true. In the example, 'If Monika arrives before 3 pm then she will be in time to see the concert', if the first part is known to be true (she arrives before 3pm), then it can be inferred that the second part must also be true (she will be in time to see the concert).

2. From conditional statement 'If x then y' and information that y is not true, it can be inferred that x must also not be true.

The statement 'If x then y' implies that if x is true, y must also be true. This means that it is not possible to have x without y. So, if y is not true, then x cannot be true either. In the example, if the information 'Monika did **not** arrive in time to see the concert' is given, it can be inferred that Monika did **not** arrive before 3 pm (the statement 'Monika arrives before 3 pm' must be false).

3. From conditional statement 'If x then y' and information that y is true, it cannot be inferred that x must also be true.

The statement 'If x then y' implies that if x is true, y must also be true. It does not imply that if y is true then x must also always be true; it could be possible to have y without x. For example, if Monika is in time to see the concert, this does not necessarily mean that she arrived before 3 pm. The concert may not start until 4 pm, so she could still be in time if she arrived at 3.30 pm.

4. From conditional statement 'If x then y' and information that x is *not* true, it *cannot* be inferred that y is *not true* either.

Similarly, the statement 'If x then y' does not provide any information about what would happen if x were **not** true because it does not imply that y can *only* be true if x is true; it could be possible to have y without x. In the example, if Monika does not arrive before 3 pm then it cannot be inferred that she was (or was not) in time to see the concert because, as before, it may be possible for her to arrive later than 3 pm and still be in time.

To summarise, there are only two valid logical deductions that can be made from conditional statements when provided with information about the truth or falsity of individual parts:

Statement	Additional information	Inference
If x then y	and x is true	then y is true
If x then y	and y not true	then x not true

Logical deductions

A logical deduction is any conclusion that can be drawn with absolute certainty from the information given. This is known as *deductive* reasoning.

Inductive reasoning

A distinction can be made between deductive reasoning, where the conclusions depend purely on logic and can be inferred with absolute certainty, and inductive reasoning, where the conclusions are more probabilistic and based more on the strength of the evidence.

In an inductive argument, the evidence is used to make the conclusion more likely.

For example, 'A class had three separate races last week. John won each race. He is clearly the fastest runner in the class.' In this case, it cannot be concluded with absolute certainty that John is the fastest runner. Perhaps there is a faster runner who was absent or injured. However, the fact that John won three races does give good evidential support to the claim that he is the fastest runner. Without any evidence, there would be no reason to think it were true. With this evidence, the claim has now become more probable.

In some cases, however, the evidence is sufficient to demonstrate that a claim is certainly false.

For example, a gym claims, 'If you attend athletics training, then you can be certain that your race time will improve.'

A member of the gym provides some evidence for accepting this claim. They say, 'Everyone I know who attended training has improved their race time dramatically.'

This is corroborating evidence for the claim and makes the claim more likely to be true.

However, someone else then says, 'Alice has been attending the training for a month and her time has got worse.'

From this last piece of evidence, and the rules about conditional statements (if the consequent of a conditional statement is false, then the antecedent is also false), it can be inferred with certainty that the claim is false.

Questions on inductive reasoning will typically ask candidates to determine what the effect is of a given piece of new information on another claim or argument.

Does it make it more or less likely to be true? Does it show that it is very probably true? Or does it have no actual effect? Or does it actually refute it altogether?

Notions of consistency and inconsistency

If two statements are *inconsistent* it means that they cannot both be true. So, at least one of them must be false.

Recognising inconsistencies allows the elimination of possibilities. If two things are inconsistent and it is known, or can be deduced, that one of the two things is true, then the other one must be false.

In the example about the athletics training, the statement 'If you attend athletics training, then you can be certain that your race time will improve' is *inconsistent* with the information that 'Alice has been attending the training for a month and her time has got worse'. If the information about Alice is true, then the original statement cannot be true.

Sample Questions

1. Three friends are at a pizza restaurant. The waiter asks them, 'Does everyone want pizza?'

The first friend says, 'I don't know.'

The second friend says, 'I don't know.'

The third friend says, 'No, not everyone wants pizza.'

Which statement is true?

- A. The first and second friends do not want pizza.
- B. The third friend does not want pizza.
- C. The first and third friends want pizza.
- D. It is undecidable.

Key: B

2. Three sisters, Emma, Sarah and Rachel, are each of different heights.

If Sarah is not the tallest, then Rachel is.

If Rachel is not the shortest, then Leah is the tallest.

Who is the tallest and who is the shortest?

- A. Sarah is the tallest and Rachel is the shortest.
- B. Rachel is the tallest and Leah is the shortest.
- C. Rachel is the tallest and Sarah is the shortest.
- D. It is not possible to determine the relative heights from the information given.

Key: A

3. A museum has 15 paintings, each of which was painted by a different artist. The curator knows that at least one painting is from the 20th Century and, given any two paintings, at least one was painted before the 19th Century.

Based on these facts, can you determine how many of the paintings were painted in the 20th Century?

- A. It is not possible to determine.
- B. 7
- C. 2
- D. 1

Key: D