Physics

1. A 5.0 kg block on a 30° incline (kinetic friction coefficient $\mu_k = 0.20$) is connected over a frictionless pulley to a hanging 4.0 kg mass. If the system is released from rest, the magnitude of the acceleration is closest to

(A) $0.20\,\mathrm{m/s^2}$

(B) $0.70\,\mathrm{m/s^2}$

(C) $1.4\,\mathrm{m/s^2}$

(D) $2.8\,\mathrm{m/s^2}$

(E) $4.9\,\mathrm{m/s^2}$

2. A solid sphere rolls without slipping down from height $h = 1.2\,\mathrm{m}$. Neglect losses. The speed at the bottom is closest to

(A) $3.0\,\mathrm{m/s}$

(B) $4.1\,\mathrm{m/s}$

(C) $4.9\,\mathrm{m/s}$

(D) $6.9\,\mathrm{m/s}$

(E) $8.4\,\mathrm{m/s}$

3. A 2.0 kg cart moving at 6.0 m/s collides and sticks to a 4.0 kg cart at rest. The combined mass then compresses a spring ($k = 300\,\mathrm{N/m}$) on a frictionless track. The maximum compression is closest to

(A) $0.10\,\mathrm{m}$

(B) $0.28\,\mathrm{m}$

(C) $0.40\,\mathrm{m}$

(D) $0.57\,\mathrm{m}$

(E) $0.80\,\mathrm{m}$

4. A point charge $+Q$ is placed at the center of a neutral conducting spherical shell (inner radius a, outer radius b). The electric field magnitude at radius r such that $a < r < b$ is

(A) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

(B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{b^2}$

(C) 0

(D) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a^2}$

(E) cannot be determined without knowing a and b

5. A parallel-plate capacitor is charged to voltage V_0 and then disconnected from the battery. A dielectric with constant $\kappa = 3$ fully fills the gap. The ratio of final to initial stored energy U_f/U_i is

(A) 3

(B) $\sqrt{3}$

(C) $1/3$

(D) $1/\sqrt{3}$

(E) $1/9$

6. A series RL circuit with $R = 20\,\Omega$ and $L = 0.50\,\text{H}$ is connected to a $10\,\text{V}$ DC source at $t = 0$. The current at $t = 0.050\,\text{s}$ is closest to

(A) $0.07\,\text{A}$

(B) $0.43\,\text{A}$

(C) $0.50\,\text{A}$

(D) $0.86\,\text{A}$

(E) $1.0\,\text{A}$

7. Two thin lenses are separated by $20\,\text{cm}$. Lens 1 has $f_1 = +10\,\text{cm}$ and lens 2 has $f_2 = -15\,\text{cm}$. An object is placed $30\,\text{cm}$ to the left of lens 1. The final image is located

(A) $3.75\,\text{cm}$ to the right of lens 2 (real)

(B) $3.75\,\text{cm}$ to the left of lens 2 (virtual)

(C) $15\,\text{cm}$ to the right of lens 2 (real)

(D) $15\,\text{cm}$ to the left of lens 2 (virtual)

(E) at infinity

8. A uniform ladder of length $5.0\,\text{m}$ and weight $200\,\text{N}$ rests against a *smooth* vertical wall and a *rough* horizontal floor. The ladder makes an angle of 60° with the floor. The minimum coefficient of static friction at the floor to prevent slipping is closest to

(A) 0.10

(B) 0.20

(C) 0.29

(D) 0.50

(E) 0.87

9. One mole of a monatomic ideal gas ($\gamma = 5/3$) is compressed adiabatically from $V_1 = 10\,\text{L}$ and $T_1 = 300\,\text{K}$ to $V_2 = 2.0\,\text{L}$. The final temperature T_2 is closest to

(A) $430\,\text{K}$

(B) $600\,\text{K}$

(C) $880\,\text{K}$

(D) $1500\,\text{K}$

(E) $3000\,\text{K}$

10. In the Bohr model of hydrogen, the photon emitted in the transition $n = 4 \to n = 2$ has wavelength closest to

(A) $122\,\text{nm}$

(B) $365\,\text{nm}$

(C) $486\,\text{nm}$

(D) $656\,\text{nm}$

(E) $972\,\text{nm}$

Mathematics

1. It is a well-known result that the Fourier transform of a Gaussian is again a Gaussian. For $f(x) = e^{-ax^2}$ with $a > 0$, which of the following has the correct functional form of the Fourier transform?

   (A) $\sqrt{\frac{\pi}{a}}\, e^{-k^2/4a}$
   (B) $\frac{1}{\sqrt{a}}\, e^{-ak^2}$
   (C) $\sqrt{\pi a}\, e^{-ak^2}$
   (D) $e^{-k^2/a}$
   (E) $\frac{\pi}{a}\, e^{-k^2/a}$

2. Evaluate the limit:
$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$
   (A) 0
   (B) $\frac{1}{2}$
   (C) 1
   (D) ∞
   (E) Does not exist

3. Which of the following is the correct expression for the volume of the solid generated by rotating the region bounded by $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$ about the y-axis (using the method of cylindrical shells)?

   (A) $\int_a^b \pi [f(x)]^2\, dx$
   (B) $\int_a^b 2\pi [f(x)]^2\, dx$
   (C) $\int_a^b 2\pi x f(x)\, dx$
   (D) $\int_a^b \pi x f(x)\, dx$
   (E) $\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

4. The eigenvalues of the matrix
$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
   are:

   (A) $\lambda = 0,\ 2$
   (B) $\lambda = 1,\ 3$
   (C) $\lambda = 2,\ 2$
   (D) $\lambda = -1,\ 3$
   (E) $\lambda = 1,\ 2$

5. Let $\mathbf{A} = (1, 2, 3)$ and $\mathbf{B} = (4, 5, 6)$. The cross product $\mathbf{A} \times \mathbf{B}$ is:

   (A) $(3, -6, 3)$
   (B) $(2, 3, 4)$

(C) $(0, 0, 0)$
(D) $(-3, 6, -3)$
(E) $(1, 1, 1)$

6. A rank-2 symmetric tensor $T_{\mu\nu}$ (i.e. $T_{\mu\nu} = T_{\nu\mu}$) in 4-dimensional spacetime has how many independent components?

   (A) 4
   (B) 6
   (C) 10
   (D) 12
   (E) 16

7. The equation $\partial_t u = \gamma \nabla^2 u$ is:

   (A) Elliptic
   (B) Hyperbolic
   (C) Parabolic
   (D) Linear algebraic
   (E) Nonlinear

8. Consider the curve defined by the equation $x^2 + y^2 = 25$. What is the slope of the tangent line to the curve at the point $(3, 4)$?

   (A) $-3/4$
   (B) $3/4$
   (C) $-4/3$
   (D) 0
   (E) 1

9. Find the x-coordinate of the inflection point for the function $f(x) = x^3 - 6x^2 + 9x + 5$.

   (A) $x = 0$
   (B) $x = 1$
   (C) $x = 2$
   (D) $x = 3$
   (E) $x = 4$

10. Evaluate the integral:

$$\int_0^\infty e^{-ax}\, dx \quad (a > 0)$$

   (A) $\frac{1}{a}$
   (B) a
   (C) $\frac{1}{a^2}$
   (D) 0
   (E) Diverges